DO NOW

5.4 The Fundamental Theorem of Calculus

Differential Calculus	vs.	Integral Calculus
$\text{Slope} = \frac{\Delta y}{\Delta x}$ (h) (w)		$\text{Area} = \overset{h \cdot w}{\Delta y \cdot \Delta x}$
derivatives		integrals

* Inverses...

The Fundamental Theorem of Calculus

If f is continuous on $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then:

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

f = function

F = Integral or Antiderivative

Examples: Evaluate the definite integral of the function.

1. $\int_{-1}^{2} (x^2 + 3)\,dx$

$\left[\frac{x^3}{3} + 3x\right]_{-1}^{2}$ use $F(2) - F(-1)$

$\left(\frac{2^3}{3} + 3(2)\right) - \left(\frac{(-1)^3}{3} + 3(-1)\right)$

$\frac{8}{3} + 6 + \frac{1}{3} + 3$

$\boxed{12}$

2. $\int_0^2 (2 - t)\sqrt{t}\,dt = \int_0^2 \left(2t^{1/2} - t^{3/2}\right) dt$

$\left[\frac{2t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2}\right]_0^2$

$\left[\frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^2$

$\rightarrow F(2) - F(0)$

$\left[\frac{4}{3}(2)^{3/2} - \frac{2}{5}(2)^{5/2}\right] - \left[\frac{4}{3}(0)^{3/2} - \frac{2}{5}(0)^{5/2}\right]$

$\frac{4}{3}\sqrt{8} - \frac{2}{5}\sqrt{32} - 0$

$\frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$\frac{40\sqrt{2} - 24\sqrt{2}}{15} = \frac{16\sqrt{2}}{15}$ ← Exact Area

Approximately 1.51

What happened to the constant 'C'???

$\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b$

$\Big[F(b) + C\Big] - \Big[F(a) + C\Big]$

$F(b) - F(a)$

3. $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \Big[\tan x\Big]_0^{\pi/4}$

$\tan \pi/4 - \tan 0$

$1 - 0$

$\boxed{1}$

4. $\int_0^2 |2x - 1|\,dx$

$2x-1=0$

$x=\frac{1}{2}$

* change signs if $x < \frac{1}{2}$

$\int_0^{1/2} -(2x-1)\,dx + \int_{1/2}^{2} (2x-1)\,dx$

$\left[-(x^2-x)\right]_0^{1/2} + \left[x^2-x\right]_{1/2}^{2}$

$\left[-x^2+x\right]_0^{1/2} + \left[x^2-x\right]_{1/2}^{2}$

$\left[\left(-(\frac{1}{2})^2+\frac{1}{2}\right)-(-0^2+0)\right] + \left[(2^2-2)-\left((\frac{1}{2})^2-\frac{1}{2}\right)\right]$

$\frac{1}{4}+2+\frac{1}{4}$

$2\frac{1}{2}$

$\frac{5}{2}$

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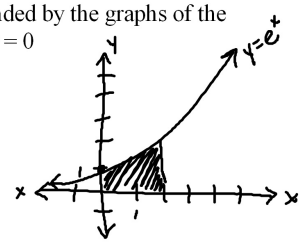
5. Find the area of the region bounded by the graphs of the equations: $y = e^x, x = 0, x = 2, y = 0$



$\int_0^2 e^x\,dx$

$\left[e^x\right]_0^2$

e^2-e^0

e^2-1 ← exact area

6.39 ← Approximate area

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HOMEWORK

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